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MASS TRANSFER FROM A SINGLE BUBBLE TO THE DENSE PHASE OF A FLUIDIZED BED AT LARGE PECLET NUMBERS

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Possible modes of mass exchange between a growing bubble and the dense phase under conditions of predominance of convective transfer are discussed.

The productivity and quality of operation of catalytic chemical reactors and of a number of other apparatus containing an inhomogeneous fluidized bed depend on the intensity of gas exchange between the dilute and dense phases of the bed. The determination of the corresponding coefficient of mass exchange, which plays a very important role in modern systems of calculation of such apparatus (see the review in [1-3], for example), represents one of the central problems in modeling them.

The number of experiments on the determination of mass-exchange characteristics is very large (there is also a summary of them in [1-3]), but generalizing correlations which would permit the construction of a complete representation of the variation of these characteristics with variation in the fluidization conditions and of ways to intensify mass exchange are practically absent. This is connected both with the variety of the mechanisms of mass transfer in two-phase systems and of the factors influencing it and with the fact that the majority of the experimental data have been obtained by indirect means (by methods of a model chemical reaction or a tracer gas) from a comparison of the observed concentrations of the reagents at the exit from the reactor or of washing curves with results following from one or another model.

A theoretical analysis of the specific roles of different mechanisms of the process of mass transfer under different conditions and possible limiting modes of realization of the process becomes especially necessary under such conditions. Very little has been done in this direction up to now. The theoretical model in [4], within the framework of which only the mass exchange of a bubble with the cloud of closed gas circulation surrounding it was allowed for, neglecting the diffusional resistance of the dense phase, evidently was the first. Unfounded assumptions of such a type were adopted later in [5-7]. Conversely, an equally unfounded assumption about the total gas mixing in the bubble and the cloud was adopted in [8, 9], and then in [10] also, and attention was concentrated on the investigation of convective diffusion in the dense phase outside the cloud. An attempt made by Kunii and Levenspiel [11] to simultaneously allow for the resistance to mass exchange both outside and inside the cloud has an especially empirical character.

It is important that in all these reports the influence of the variation of actual bubbles as they rise in a bed on the mass exchange was entirely ignored. The volume of bubbles which are not too small grows in the process, i.e., there is a gas flux directed into the bubble. The convective transfer by this flux must hinder the removal of an impurity into the dense phase and, conversely, facilitate its penetration into the bubble, which is confirmed, in particular, by the tests in [12, 13], conducted on a plane model of a bed containing "two-dimensional" bubbles. The necessity of allowing for the influence of the growth of a bubble on its mass exchange with the dense phase was noted in [14, 15], where a theoretical analysis of the motion and growth of a bubble in a developed fluidized bed was proposed. Experimental data on bubble growth obtained by various authors are discussed in [16, 17], where empirical relations are proposed for the description of this effect.

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In order to avoid the additional difficulties associated with the need to analyze the hydrodynamic interaction in a system of many bubbles and to allow for the variation in the state of the dense phase over the height of the bed, below we will consider only the mass transfer from a single bubble to a dense phase whose characteristics are homogeneous. For a similar reason we neglect here the influence on the mass transfer of an impurity of homo- and heterogeneous chemical reactions proceeding with its participation, as well as its transfer by moving particles on whose surface it is capable of being adsorbed. In addition, as in [4-11], we assume that the velocity of rise of the bubble exceeds the velocity of gas filtration in the dense phase, i.e., we consider only bubbles which would possess a cloud with closed circulation if their volumes remained constant. For simplicity, we take the bubble and the region of closed circulation as spherical and analyze the transfer of gas through the outer boundary of this region.

The true gas flow near a bubble can be represented approximately as a superposition of two flows: that generated by flow over a bubble of constant radius and the radial flow directed toward the bubble and leading to its growth. We describe the first of these by the stream function [4]

$$\psi = (U-u)\left(1-\frac{R_c^3}{r^3}\right)\frac{r^2}{2}\sin^2\theta, \quad \left(\frac{R_c}{R_b}\right)^3 = \frac{U+2u}{U-u}. \tag{1}$$

Here we use a spherical coordinate system connected with the center of the bubble; the point of onflow of the stream corresponds to the value $\theta = \pi$ of the polar angle. The velocity components u_r and u_θ of this flow are obtained from (1) in the usual way.

To describe the radial flow we adopt the empirical dependence of the radius of a bubble on its vertical coordinate h in the bed obtained in [17]:

$$R_b \approx 0.93\,(\varepsilon u - u_*)^{2/3} h^{2/3} g^{-1/3}. \tag{2}$$

Using the Davis–Taylor equation

$$dh/dt = U \approx 0.71\,(2gR_b)^{1/2} \approx (gR_b)^{1/2} \tag{3}$$

for the velocity of rise of a bubble, from (2) we obtain the expression for the velocity of radial flow:

$$-v_r = v = v_0\left(\frac{R_b}{r}\right)^2, \quad \varepsilon v_0 = \frac{dR_b}{dt} = \frac{dR_b}{dh}\,U \approx 0.6\,(\varepsilon u - u_*). \tag{4}$$

We write the equations of convective diffusion outside and inside the cloud in the form

$$\varepsilon\left(\frac{\partial}{\partial t} + (u_r + v_r)\frac{\partial}{\partial r} + \frac{u_\theta}{r}\frac{\partial}{\partial\theta}\right)\begin{Bmatrix} c \\ c' \end{Bmatrix} = \nabla\left(\begin{Bmatrix} \varepsilon D \\ \varepsilon D' \end{Bmatrix}\nabla\begin{Bmatrix} c \\ c' \end{Bmatrix}\right) \tag{5}$$

(quantities with a prime refer to the region inside the cloud). The effective coefficients of diffusion describe the molecular diffusion with allowance for a sinuosity factor and the convective dispersion due to random pseudoturbulent pulsations of the gas velocity (see [18]). The corresponding coefficients of dispersion depend on the local value of the filtration velocity relative to the particles, which depends both on the coordinates (in view of the nonuniformity of the flow field) and on time (in view of the time dependence of the rise velocity of the growing bubble). Therefore, the coefficients D and D' in (5) also depend on time and the coordinates in general. Because of the assumption that the dense phase is homogeneous, these coefficients must be given by the same functional equations outside and inside the cloud. However, D' differs considerably from D inside the bubble itself, which is practically free of particles, where D' approximately coincides with the coefficient of molecular diffusion.

The solution of Eq. (5) with coefficients dependent on the coordinates and time and with definite initial and boundary conditions represents a very complex task, even for a single bubble, and one can be confident of obtaining forseeable analytical equations for the gas flux through the outer boundary of the cloud, the coefficient of mass exchange, etc., only in the simplest limiting cases under a series of additional simplifying assumptions. First of all, we assume that the Peclet number is large, so that it dominates the convective gas transfer. In this case the concentrations outside and inside the cloud differ from the corresponding homogeneous values only in thin layers with thicknesses δ and δ' on both sides of the sphere $r = R_c$. The terms containing u_r and u_θ in (5) have the same order of magnitude within the limits of these layers, and it follows from (1) that $u_r \sim (U - u)\delta/R_c$. Using the expression for v_r from (4), we can see that in the very probable case when the inequalities

$$(U-u)\delta \ll (\varepsilon u - u_*) R_c, \quad (U-u)\delta' \ll (\varepsilon u - u_*) R_c \tag{6}$$

occur, we can neglect the influence of the stream flowing over the cloud on the transfer of the impurity at its boundary in comparison with the influence of the radial flow. Conversely, when the inequalities opposite to (6) are satisfied the influence of the radial flow on the formation of the diffusional layers is unimportant. First of all, we must consider the limiting modes corresponding to the fulfillment of the inequalities (6) or of the strong inequalities opposite to them. In doing this we can assume, in view of the thinness of the diffusional layers, that the inner layer lies entirely inside the cloud, and we make no distinction between D and D'.

In obtaining order-of-magnitude functions it is natural to treat the problem in a "quasi-steady" formulation, neglecting the time dependence of the velocities, and thereby of the coefficients of diffusion. Moreover, we neglect the dependence of $D = D'$ on the coordinates, treating D as an effective coefficient characterizing the molecular diffusion and convective dispersion "on the average." We note that this assumption has been made in all the reports on the mass exchange of bubbles known to the authors, in [4-11], in particular. For sufficiently large bubbles the time dependence of these quantities is actually quite weak; a dependence of the coefficient of diffusion on the coordinates leads, generally speaking, to a number of new effects, discussed in [19] on the example of another problem.

Let inequalities (6) be satisfied. Then, changing to the coordinate system (ρ, θ), where $\rho = r - R_c$, assuming that the velocity v_r hardly depends on the coordinates within the limits of the diffusional layers, and retaining only the principal term on the right side of (5), we obtain

$$\left(\frac{\partial}{\partial t} - 2V\frac{\partial}{\partial \rho}\right)\begin{Bmatrix} c \\ c' \end{Bmatrix} = D\frac{\partial^2}{\partial \rho^2}\begin{Bmatrix} c \\ c' \end{Bmatrix}, \quad V = v_0\left(\frac{R_b}{R_c}\right)^2, \quad \rho = r - R_c. \tag{7}$$

We seek the solution of (7) with the conditions

$$\begin{gathered} c = c_0,\ t = 0,\ \rho > 0;\ c = c_0',\ t = 0,\ \rho < 0;\ c \to c_0,\ \rho \to \infty; \\ c' \to c_0',\ \rho \to -\infty;\ c = c',\ \partial c/\partial \rho = \partial c'/\partial \rho,\ \rho = 0. \end{gathered} \tag{8}$$

The boundary conditions in (8) in the approximation of a thin diffusional layer are obvious, while the initial conditions correspond to the idea that both the bubble and the cloud surrounding it, appearing at the time $t = 0$, are filled with a gas with an impurity concentration c_0' different from its concentration c_0 in the dense phase far from the bubble.

Using a Laplace transformation with the parameter p, we obtain the following expressions for the transforms of the unknown functions:

$$\begin{gathered} \hat{c}(p) = \frac{c_0}{p} + \frac{1}{2}\frac{c_0' - c_0}{p}\left(1 - \frac{V}{\lambda D}\right)\exp\left[-\rho\left(\lambda + \frac{V}{D}\right)\right],\ \rho > 0, \\ \hat{c}'(p) = \frac{c_0'}{p} - \frac{1}{2}\frac{c_0' - c_0}{p}\left(1 + \frac{V}{\lambda D}\right)\exp\left[\rho\left(\lambda - \frac{V}{D}\right)\right],\ \rho < 0, \\ \lambda = \left[\frac{p}{D} + \left(\frac{V}{D}\right)^2\right]^{1/2}. \end{gathered} \tag{9}$$

The corresponding inverse transforms have the form

$$\begin{gathered} c = c_0 + \frac{c_0' - c_0}{2}\operatorname{erfc}\left[\frac{\rho}{2(Dt)^{1/2}} + \left(\frac{V^2 t}{D}\right)^{1/2}\right],\ \rho > 0, \\ c' = c_0' - \frac{c_0' - c_0}{2}\operatorname{erfc}\left[-\frac{\rho}{2(Dt)^{1/2}} - \left(\frac{V^2 t}{D}\right)^{1/2}\right],\ \rho < 0. \end{gathered} \tag{10}$$

The flux through the outer boundary of the cloud in the case when it is directed into the cloud ($c_0 > c_0'$) is

$$q = \varepsilon V\left[c_0 + \frac{c_0 - c_0'}{2}\left(\frac{1}{\sqrt{\pi\tau}}e^{-\tau} - \operatorname{erfc}\sqrt{\tau}\right)\right] \approx \varepsilon V\left(c_0 + \frac{c_0 - c_0'}{4\tau\sqrt{\pi\tau}}e^{-\tau}\right). \tag{11}$$

When $c_0' > c_0$ the flux directed from the cloud into the surrounding medium is

$$q = \varepsilon V\left[-c_0 + \frac{c_0' - c_0}{2}\left(\frac{1}{\sqrt{\pi\tau}}e^{-\tau} + \operatorname{erfc}\sqrt{\tau}\right)\right] \approx \varepsilon V\left(-c_0 + \frac{c_0' - c_0}{\sqrt{\pi\tau}}e^{-\tau}\right). \tag{12}$$

Here we introduce the dimensionless time

$$\tau = V^2 t/D. \tag{13}$$

The fluxes in (11) and (12) refer to a unit surface area of the region of closed circulation; the approximate equalities correspond to asymptotic forms at $\tau \gg 1$. The values of the fluxes referred to a unit surface area of the bubble itself are obtained by replacing V by v_0 in (11) and (12). It is undesirable to introduce the Sherwood number to characterize the mass-exchange process in the present case. Since the distribution of the gas inflow velocity through the surface of the bubble or cloud is uniform, the distributions of the fluxes of the impurity in (11) and (12) are also uniform.

It is easy to see from (12) that the flux initially directed from the bubble into the dense phase changes its sign with time. Such a "cutoff" of the flux from the bubble occurs at the time $\tau = \tau_*$, where τ_* is the only root of the equation

$$\frac{c_0}{c_0' - c_0} = \frac{1}{\sqrt{\pi\tau_*}}e^{-\tau_*} + \operatorname{erfc}\sqrt{\tau_*} \approx \frac{2}{\sqrt{\pi\tau_*}}e^{-\tau_*}. \tag{14}$$

At times $\tau > \tau_*$ the gas impurity which is inside the region of closed circulation proves to be "conserved" — it is no longer capable of penetrating into the interior of the dense phase of the bed.

In the present case the quantity $Pe_1 = VR_c/D$ plays the role of the Peclet number. From (10) we get estimates for the thicknesses of the diffusional layers,

$$\delta \sim (Dt)^{1/2}, \quad \delta' \sim Vt, \tag{15}$$

with the concentrations in both layers differing very weakly from c_0 at $\tau \gg 1$. It is seen from (15) that this mode is realized at the initial times even when $Pe_1 \lesssim 1$. The conditions for its realization are violated at a time equal in order of magnitude to the lesser of the times

$$T = \frac{R_c^2}{D}\left(\frac{\varepsilon u - u_*}{U - u}\right)^2, \quad T' = \frac{R_c}{V}\,\frac{\varepsilon u - u_*}{U - u}. \tag{16}$$

The ratio of these times is

$$\frac{T'}{T} = \frac{1}{Pe_1}\,\frac{U - u}{\varepsilon u - u_*}, \qquad Pe_1 = \frac{R_c V}{D}. \tag{17}$$

If $T' < T$, which is possible at large Pe_1 for bubbles which are not too large in beds of large particles having a dense phase whose porosity differs considerably from that in the state of minimum fluidization, then the second inequality in (6) is violated first. In this case the assumption of total mixing in the dense phase, which was adopted in [4-7], is legitimate during some time interval between T and T'. If $T < T'$ then the first inequality in (6) is violated first, and at $t \gtrsim T$ the structure of the outer diffusional layer will be determined not only by the radial flow (4) but also by the flow (1). However, the assumption of ideal mixing inside the region of closed circulation, which was adopted in [8-10], is inadequate in the present case, since the concentration in the inner layer is close to c_0 rather than to c_0'. At $t \gtrsim \max\{T, T'\}$ the flow with the stream function (1) becomes important in the formation of both diffusional layers.

We note that the mode discussed above is essentially nonsteady; moreover, a solution of the steady-state analog of the problem (7), (8) is entirely absent.

Now let us consider the opposite limiting case, when the strong inequalities opposite to (6) are satisfied and the flow (1) has the dominant influence on the formation of both diffusional layers. The establishment of a steady mode of mass exchange as a limit to which the true nonsteady mode approaches with an increase in the thickness (15) of these layers is evidently possible in this case. (We emphasize, however, that to prove that such a mode is realized in practice, at least in a very deep bed, one must, of course, examine the evolution of the mass-exchange process for different initial values of the parameter.)

In the indicated case the steady-state problem for the concentrations c and c' can be written in the form (see [20], for example)

$$\left(u_r \frac{\partial}{\partial \rho} + u_\theta \frac{1}{R_c} \frac{\partial}{\partial \theta}\right) \begin{Bmatrix} c \\ c' \end{Bmatrix} = D \frac{\partial^2}{\partial \rho^2} \begin{Bmatrix} c \\ c' \end{Bmatrix}, \tag{18}$$

$$\begin{array}{ll} c \to c_0,\ \rho \to \infty;\ c' \to c_0',\ \rho \to -\infty; & \begin{Bmatrix} c \\ c' \end{Bmatrix} = \begin{Bmatrix} c_0 \\ c_0' \end{Bmatrix};\ \theta = \pi. \\ c = c',\ \partial c/\partial \rho = \partial c'/\partial \rho,\ \rho = 0; & \end{array}$$

The two conditions at $\theta = \pi$ correspond to the well-known condition at the point of onflow of the stream, which has been used in [20] and other reports.

Introducing the new variables ψ and ξ in the standard way, where the stream function ψ is defined in (1) while ξ is given by the equation

$$\xi = \frac{1}{2}(U-u) D R_c^3 (2 + 3\cos\theta - \cos^3\theta), \tag{19}$$

we convert the problem (18) to the form

$$\frac{\partial}{\partial \xi} \begin{Bmatrix} c \\ c' \end{Bmatrix} = \frac{\partial^2}{\partial \psi^2} \begin{Bmatrix} c \\ c' \end{Bmatrix}, \tag{20}$$

$$\begin{array}{ll} c \to c_0,\ \psi \to \infty;\ c' \to c_0',\ \psi \to -\infty; & \begin{Bmatrix} c \\ c' \end{Bmatrix} = \begin{Bmatrix} c_0 \\ c_0' \end{Bmatrix},\ \xi = 0. \\ c = c',\ \partial c/\partial \psi = \partial c'/\partial \psi,\ \psi = 0; & \end{array}$$

The solution of the problem (20) has the form

$$c = c_0 + \frac{c_0' - c_0}{2} \operatorname{erfc} \frac{\psi}{2\sqrt{\xi}},\quad c' = c_0' + \frac{c_0 - c_0'}{2} \operatorname{erfc} \frac{-\psi}{2\sqrt{\xi}}, \tag{21}$$

while the total flux of the gaseous impurity through a unit surface area of the region of closed circulation is represented in the form

$$q = \pm q^C + q^D,\quad q^C = \varepsilon V \frac{c_0 + c_0'}{2},\quad q^D = \varepsilon D \frac{|c_0 - c_0'|}{\delta + \delta'}, \tag{22}$$

where the upper and lower signs refer to fluxes into and out of the bubble, respectively, while the thicknesses of the diffusional layers are

$$\delta = \delta' = \frac{\sqrt{2\pi}}{3} \left(\frac{R_c D}{U-u}\right)^{1/2} \frac{(2 - \cos\theta)^{1/2}}{1 - \cos\theta}. \tag{23}$$

The total flux is obtained after integration of (22) over the sphere $r = R_c$; we have $Q = \pm Q^C + Q^D$, where

$$Q^C = 4\pi\varepsilon \frac{c_0 + c_0'}{2} R_c^2 V = 4\pi\varepsilon \frac{c_0 + c_0'}{2} \left(\frac{U + 2u}{U - u}\right)^{2/3} R_b^2 V;$$

$$Q^D = 4\sqrt{\frac{\pi}{2}}\, \varepsilon |c_0 - c_0'| [(U-u) D]^{1/2} R_c^{3/2} = 4\sqrt{\frac{\pi}{2}}\, \varepsilon |c_0 - c_0'| [(U + 2u) D]^{1/2} R_b^{3/2}. \tag{24}$$

The quantity $Pe_2 = R_c U/D$ plays the role of the Peclet number, which must be large in comparison with unity for the adequacy of the approximation of a thin diffusional layer used, The ratio of the total flux components (24) is important:

$$\frac{Q^D}{Q^C}=\sqrt{\frac{2}{\pi}}\left|\frac{c_0-c_0'}{c_0+c_0'}\right|\frac{1}{\sqrt{\mathrm{Pe}_2}}\left(1-\frac{u}{U}\right)^{1/2}\frac{U-u}{V},\ \mathrm{Pe}_2=\frac{R_cU}{D}. \tag{25}$$

It is seen that these components are comparable in magnitude, generally speaking, since the parameter $(U-u)/V$ in (25), which can be large, is multiplied by the small quantity $\mathrm{Pe}_2^{-1/2}$. Therefore, it is inadmissible in general to neglect the convective flux component Q^C. Only in the case when the state of the dense phase is close to that reached at minimum fluidization can one take $Q \approx Q^D$ and introduce the Sherwood number

$$\mathrm{Sh}\approx\frac{Q}{4\pi R_c\varepsilon D\,|c_0-c_0'|}=\frac{1}{\sqrt{2\pi}}\left(\frac{R_bU}{D}\right)^{1/2}\left(1+\frac{2u}{U}\right)^{1/6}\left(1-\frac{u}{U}\right)^{1/3}, \tag{26}$$

which proves to be twice as small as that calculated in [10]. (The error in [10] is connected with the use of the unjustified assumption of total mixing inside the region of closed circulation.)

It is easy to see that when the flux of the impurity is directed from the bubble into the dense phase a change in the sign of the flux is also possible, with the "cutoff" condition analogous to (14) being

$$Q^D/Q^C=1 \tag{27}$$

in this case. It is seen that if $Q = Q^C + Q^D > 0$ at some time (i.e., at some level in the bed), then the condition (27) can be fully satisfied at some subsequent time (i.e., at a higher level). In particular, for large bubbles, when $U - u \approx U$, we find that the removal of the impurity into the dense phase ceases when the bubble radius reaches the critical value (we use (3), (4), and (7))

$$R_*\approx 3.13\left|\frac{c_0-c_0'}{c_0+c_0'}\right|^4\frac{gD^2}{(\varepsilon u-u_*)^4}. \tag{28}$$

The quantity $V \sim \varepsilon u - u_*$, which depends on the degree of departure of the state of the dense phase from that reached in the state of minimum fluidization, played a notable role in the analysis presented above. This departure, due, for example, to certain differences of actual fluidized systems from the ideal one corresponding to the requirements of the two-phase theory of fluidization, can be quite significant. For example, with the introduction of the gas in jets through nozzles or the openings of a perforated gas-distribution grid, about half of the excess gas volume (above that required for initial fluidization) initially enters precisely into the dense rather than the dilute phase [4, 21, 22]. The porosity of the dense phase is appreciably higher than the porosity of the bed in the close-packed state and with uniform gas distribution so long as the fluidization number is not too close to unity [23]. Therefore, the influence of the convective flux due to the growth of a bubble on the mass exchange is always important, at least in the lower part of the bed.

As it is easy to see, the mass-exchange intensity per unit area of the mass-exchange surface decreases rapidly with an increase in the height of the bubble in the bed, with the rate of this decrease declining monotonically. Dependences of this type are usually observed in experiments both with single and with multiple bubbles (see [24], for example). Moreover, if the impurity enters the bed with the bubbles, then the complete cessation of its removal into the dense phase is possible as they rise in the bed. On the whole, the quantities characterizing the mass-exchange intensity in this case prove to be far lower under actual conditions than the values which would be obtained by neglecting the convective flux connected with bubble growth. This enables one to understand to some extent the fact that the coefficients of mass exchange obtained, for example, in experiments with chemical reactors are considerably lower than those estimated theoretically on the basis of approximate models in which bubble growth is ignored, as was noted more than once by the authors of [24].

Further, for a bed of large particles, a pseudoturbulent convective dispersion predominates in the diffusional transfer, and one can write $D \sim (u - u_*)^2\,\Delta t$, where Δt is the lifetime of the pseudoturbulent pulsations. The quantity V is proportional to $\varepsilon u - u_*$, so that when the porosity of the dense phase is fixed it is approximately proportional to u or u_*. Therefore, the multiplier in the definition of the dimensionless time τ in (13) depends weakly on u or u_*, and hence on the particle size, and the dependence of q on these quantities in (11) and (12) is determined by the multiplier V. This allows one to understand at once the observed increase in the mass-exchange intensity with an increase in the particle size of a bed [25-27]. A similar conclusion can also be drawn from an examination of Eqs. (24) for the second limiting mode. For a bed of fine particles D is proportional to the coefficient of molecular diffusion, whereas $V \sim \varepsilon u \to u_*$ as before. Therefore, a fixed t will correspond to a value of τ which decreases rapidly with a decrease in particle size, which leads to a sharp increase in q, as seen from (11) and (12), which is more than adequate to compensate for the corresponding decrease in q owing to the presence of the multiplier V. Consequently, for fine particles one should expect a decrease in the mass-exchange intensity with an increase in particle size; an intensity minimum should be observed at some intermediate size. The same conclusion can be drawn from an analysis of (22)-(24) in the case when the flux of the impurity is directed into the dense phase. This is qualitatively confirmed by tests conducted at the Institute of Heat and Mass Exchange, Academy of Sciences of the Belorussian SSR, some of which are described in [27].

We have not allowed for the variation of the state of the dense phase over the height of the bed due to the outflow of gas from it into the large number of rising bubbles. This effect should evidently promote a faster exchange of the modes of mass exchange due to the decrease in the effective value of V. It is obvious that to allow for this effect it is entirely necessary to consider the evolution of a collection of bubbles and of the dense phase from some initial state, assigned from the results of [21-23], for example, with allowance for the interaction and coalescence of the bubbles, which considerably complicates the problem, very complicated even without it, of the mass exchange of a single bubble with the dense phase, which was analyzed above only in the simplest cases.

NOTATION

c, c', concentrations of impurity; εD, effective coefficient of diffusion; h, vertical coordinate in bed; Q, q, total flux of impurity and flux normalized to a unit area of the mass-exchange surface; Q^C, Q^D, fluxes connected with convective and diffusional transfer; R_c, R_b, R_*, radii of region of closed circulation and of bubble and critical value of radius defined in (28); r, radial coordinate; T, T', characteristic times from (16); t, time; U, εu, u_*, velocities of bubble rise, of filtration in the dense phase, and of minimum fluidization, respectively; u_r, u_θ, velocity components of flow over a bubble of constant size; V, v_0, velocities defined in (7) and (4); δ, δ', thicknesses of diffusional layers; θ, polar angle; λ, parameter in (9); ξ, variable defined in (19); $\rho = r - R_c$; τ, dimensionless time; τ_*, root of Eq. (14); ψ, stream function from (1); quantities with a prime refer to the region of closed circulation.

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THERMAL REGIME OF MOIST CONCRETE WALLS SUBMERGED INTO THE GROUND OF STRUCTURES UNDER CONDITIONS OF CONVECTIVE DRYING

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The article provides the solution of the heat conduction for a semibounded massif with the boundary condition of the third kind taking into account the effect of evaporation on the heat-exchange surface.

Drying is a complex process of non-steady-state heat and moisture exchange which, according to the analytical theory [1, 2], is described by the system of differential equations

$$\frac{\partial t}{\partial \tau} = a\nabla^2 t + \zeta \frac{r}{c}\,\frac{\partial u}{\partial \tau},$$
$$\frac{\partial u}{\partial \tau} = a_m\nabla^2 u + a_m\delta\nabla^2 t. \qquad (1)$$

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